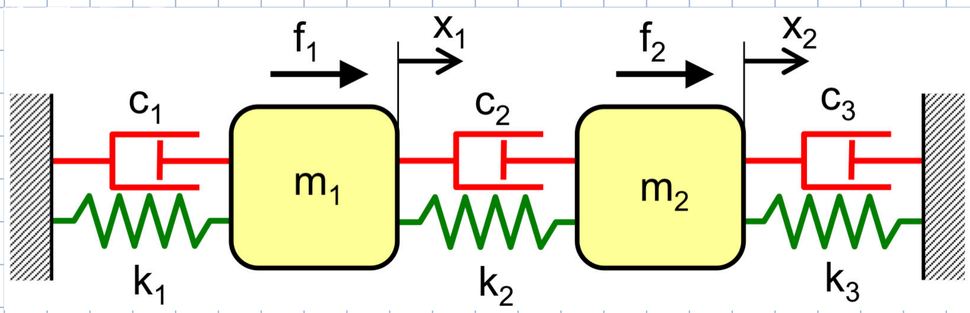
Damped 2DoF System :



Equations of dynamic equilibrium :

$$-k_1x_1 - c_1\dot{x}_1 - m_1\ddot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) + f_1 = 0$$

$$-k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1) - m_2\ddot{x}_2 - k_3x_2 - c_3\dot{x}_2 + f_2 = 0$$

In matrix form :

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \boxed{\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2+c_3 \end{bmatrix}}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

damping matrix

$$\rightarrow \quad M\ddot{x} + C\dot{x} + kx = f(t)$$

For steady-state harmonic response and excitation :

$$f(t) = \begin{bmatrix} f_{0,1} \\ f_{0,2} \end{bmatrix} \cos(\omega t) + i\sin(\omega t) = f_0 e^{i\omega t} \qquad (1)$$

$$x(t) = \begin{bmatrix} x_{0,1} \\ x_{0,2} \end{bmatrix} \cos(\omega t) + i\sin(\omega t) = x_0 e^{i\omega t} \qquad (2)$$

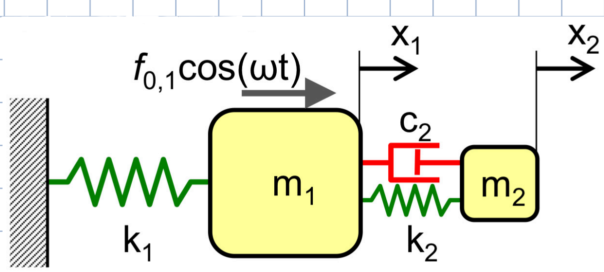
$$\hookrightarrow \quad \dot{x}(t) = i\omega x_0 e^{i\omega t}, \qquad \ddot{x}(t) = -\omega^2 x_0 e^{i\omega t}$$

Substituting (1) & (2) into EoM :

$$(-\omega^2 M + i\omega C + K)\, x_0 = f_0$$

$$\therefore \quad x_0 = (-\omega^2 M + i\omega C + K)^{-1} f_0$$

Damped TVA :



EOM in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_{0,1} \\ 0 \end{bmatrix} e^{i\omega t}$$

taking assumed solution (as in previous page) and substituting into EoM:

$$\begin{bmatrix} (k_1+k_2-\omega^2 m_1) & -k_2 - i\omega c_2 \\ -k_2 - i\omega c_2 & (k_2-\omega^2 m_2) + i\omega c_2 \end{bmatrix}\begin{bmatrix} x_{0,1} \\ x_{0,2} \end{bmatrix} = \begin{bmatrix} f_{0,1} \\ 0 \end{bmatrix}$$

Taking inverse for matrix of mass motions:

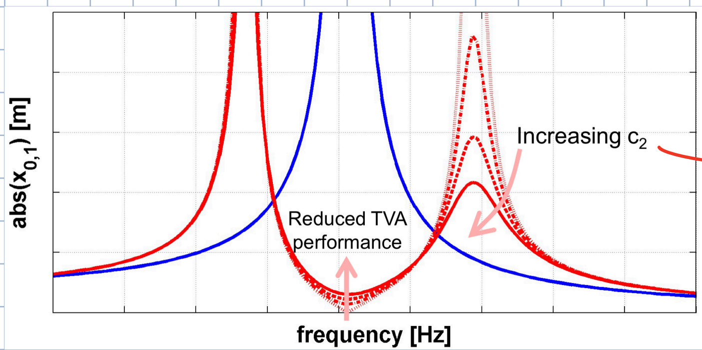
$$\begin{bmatrix} x_{0,1} \\ x_{0,2} \end{bmatrix} = \frac{1}{\det(\ldots)}\begin{bmatrix} (k_1+k_2-\omega^2 m_1) & -k_2 - i\omega c_2 \\ -k_2 - i\omega c_2 & (k_2-\omega^2 m_2) + i\omega c_2 \end{bmatrix}\begin{bmatrix} f_{0,1} \\ 0 \end{bmatrix}$$

$$\longrightarrow \quad x_{0,1} = \frac{(k_2-\omega^2 m_2) + i\omega c_2}{\det(\ldots)} f_{0,1}$$

→ for $c_2 \neq 0$, numerator cannot $= 0$

∴ damped TVA cannot be used to achieve 0 vibration for system.

Example:



→ ↑ c_2 reduces amplitude of new resonant frequencies but reduces performance of TVA in anti-resonance

↳ compromise